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Interfaces with Other Disciplines

Metatechnology frontier and convexity: A restatement

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ABSTRACT

This article reconsiders the way metafrontiers and associated measures of efficiency are obtained from nonparametric estimates of underlying group-specific frontiers. Both convex and non-convex metasets have been applied, but the large majority of articles applying this popular methodology assume that the metafrontier envelops a convex metaset. We argue that associated estimates of efficiency are potentially unreliable. We develop a refined methodology for nonparametric envelopment of non-convex metasets. We apply our methodology to a secondary data set to illustrate the potential errors associated with the currently established methods. Anticipating our main conclusion, we find that the convexification strategy consisting in assuming a convex metaset generally leads to erroneous results.

1. Introduction

Organisations in different industries, regions and countries can face different production possibilities at different points in time. Differences in so-called production possibilities sets may be due to differences in available technologies (i.e., differences in the methods that are available to transform inputs into outputs) and/or to differences in production environments (e.g., geography, climate, economic infrastructure). This article is concerned with one particular method for accounting for this type of heterogeneity when estimating production relationships.

The problem of accounting for heterogeneity when estimating production relationships is quite old. One solution that was initiated by Hayami & Ruttan (1970a) involves estimating some type of meta-production function. This meta-production function concept has been empirically applied in several agricultural studies comparing mainly country-level data (e.g., Binswanger, Yang, Bowers & Mundlak, 1987; Lau & Yotopoulos, 1989, among others). An empirical survey of this literature is provided by Trueblood (1989).

Hayami & Ruttan (1970a, p. 898) "call the envelope of all known and potentially discoverable activities a secular or 'meta-production function'." This secular production function, which is distinct from a long run production function, gives the maximum output possible using given inputs and a given amount of (existing and potentially discoverable) technical knowledge. It is implicitly assumed that all firm managers have access to the same set of input-output combinations, but each may choose a different input–output combination from that set depending on specific circumstances (i.e., government regulations, relative input prices, etc.). Cost-minimising adjustments in input mixes in response to changes in relative input prices, for example, can be viewed as movements along a secular isoquant. Trueblood (1989, Figure 1) and Hayami & Ruttan (1970b, Figure 5) draw figures depicting a secular isoquant enveloping a series of less elastic isoquants. At least part of this literature allows for inefficiency (e.g., Lau & Yotopoulos, 1989).

More recently, these basic ideas have been refined and transposed into a stochastic production frontier framework by Battese & Rao (2002) and Battese, Rao & O'Donnell (2004). The seminal article refining the loose ends in the methodology and finalising the formal framework for making efficiency comparisons across groups of firms using both stochastic frontier analysis and nonparametric deterministic frontier analysis is O'Donnell, Rao & Battese (2008). These authors consider a meta-production possibility set (or metaset for short) that is defined as the union of two or more underlying group-specific sets. They refer to the boundary of the metaset as a metafrontier, and they refer to the boundaries of the group-specific sets as group-specific frontiers (or simply group frontiers).

Thereafter, this so-called metafrontier approach has been widely applied across sectors and disciplines. Nonparametric metafrontier models have been estimated for sectors varying from agriculture (e.g., Chen & Song, 2008; Latruffe, Fogarasi & Desjeux, 2012) to banking (e.g., Casu, Ferrari & Zhao, 2013; Kontolaimou &

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Tsekouras, 2010), fisheries (Lee & Midani, 2015), hotels (Assaf, Barros & Josiassen, 2012; Huang, Ting, Lin & Lin, 2013, among others), schools (e.g., Thieme, Prior & Tortosa-Ausina, 2013), water utilities (e.g., De Witte & Marques, 2009) and wastewater treatment plants (e.g., Sala-Garrido, Molinos-Senante & Hernández-Sancho, 2011). Empirical metafrontier studies based on stochastic frontier analysis have been presented by Bos & Schmiedel (2007), Lee & Hwang (2011) and Moreira & Bravo-Ureta (2010), among others.

Meanwhile, this basic metafrontier framework has been extended in several directions: one example is the transposition to a cost (rather than production) frontier framework (e.g., Huang & Fu, 2013); another example is the estimation of the popular Malmquist productivity indices relative to metafrontiers (see, e.g., Casu et al. (2013) or Oh & Lee (2010) for a primal index and Huang, Juo & Fu (2015) for a dual approach); a final example is the introduction of more elaborate metafrontier efficiency decompositions (see Kounetas, Mourtos & Tsekouras, 2009; Tsekouras, Chatzistamoulou & Kounetas, 2017).[1]

Note that some work in the literature does not refer explicitly to the metafrontier framework, but implicitly borrows the basic idea of an overall frontier defined as the envelope of different system or group-specific frontiers. Examples include Cooper, Seiford & Tone (2007, Section 7.5) who talk about combining different "systems" and Kittelsen et al. (2015) who pragmatically define a common frontier over several Nordic countries when comparing hospital productivity.

Reliable estimates of the metafrontier allow researchers to compute reliable estimates of performance measures (e.g., technical efficiency, productivity change). In practice, it is common to use assumptions about production possibilities sets to frame the estimation of the metafrontier. Basic nonparametric frontier models are most often underpinned by the assumption that all production possibilities sets are convex (C). Convexity of a production possibilities set means that if two input vectors x_1 and x_2 can produce two output vectors y_1 and y_2, respectively, then any positive linear combination $\alpha x_1 + (1-\alpha)x_2$ with $\alpha \in [0, 1]$ of these input vectors can produce the output vector $\alpha y_1 + (1-\alpha)y_2$. The convexity assumption is usually justified using a time divisibility argument (see Shephard (1970, p. 15) and Hackman (2008, p. 39)): the argument is that if production processes are time divisible, then a manager could use x_1 to produce y_1 a proportion α of the time, and then use x_2 to produce y_2 the rest of the time. There are two weaknesses in this argument. First, the argument is untenable in the case of production processes with positive setup times (e.g., in some manufacturing processes). Second, even if group specific sets are convex, then the metaset defined by their union need normally not be C (see O'Donnell et al., 2008).[2]

Despite this mathematical fact that even convex group specific sets do not lead to a C metaset (defined as their union), O'Donnell et al. (2008) suggest estimating the metafrontier as the nonparametric boundary of a C metaset. Exactly the same convexification strategy is usually made when estimating parametric metafrontiers (e.g., Battese & Rao, 2002; Battese et al., 2004; O'Donnell et al., 2008). Since this convexification strategy is normally not true, then estimates of the metafrontier will be biased. The basic question addressed in this article is to which extent this convexification strategy yields estimates that are close to the true non-convex metaset defined as the union of group specific sets. If this approximation is close, then there is not much of a problem. However, if this approximation is poor, then potentially all articles that have so far adopted a convexification strategy when applying the metafrontier approach are potentially wrong.[3] Therefore, associated measures of firm performance (e.g., technical efficiency, productivity change) will be unreliable. This could potentially undermine the credibility of policies (e.g., price cap (or RPI-X) regulation) where these performance measures are used.

To the best of our knowledge, these issues have not been fully investigated in the literature. De Witte & Marques (2009) and Thieme et al. (2013), for example, consider nonparametric estimation of non-convex (NC) group specific-sets and metasets; they do not investigate the possibility that these sets may be convex. O'Donnell, Fallah-Fini & Triantis (2017) consider nonparametric estimation of C group-specific sets and nonconvex metasets; they do not investigate the possibility that group-specific sets may be nonconvex. Tiedemann et al. (2011) consider nonparametric estimation of C group-specific sets and NC metasets; they do not investigate the possibility that metasets may be C (see also Sala-Garrido et al., 2011). None of these authors conduct statistical tests concerning the effect of the convexification strategy on estimated efficiency scores.

Therefore, the main objectives of this article are three-fold. First, we recall some existing results and state some new results for general sets showing the potential bias of the "convexification" strategy that is used in the mainstream literature. To sharpen the focus, we consider cases where the metaset is the union of both C and NC group-specific sets. Second, to demonstrate the potential biases of the convexification strategy, we focus on the nonparametric frontier approach, since there the proper methodology to create the metafrontier from group-specific frontiers is easiest to establish (e.g., transposing and extending results from Afsharian & Ahn (2015), among others). We thereby limit the discussion to the basic C and NC nonparametric frontier specifications of technologies. The transposition of our results to alternative nonparametric and other estimators is mentioned in the concluding section. Third, using this nonparametric frontier approach, the similarities and differences between both C and NC group-specific sets and the resulting different NC metasets are empirically illustrated.

To achieve these objectives, this article is structured as follows. Section 2 explains how technologies can be represented using technology-specific production possibility sets and distance functions. Section 3 then explains how sets of technologies can be represented using metatechnology-specific production possibility sets and distance functions. Section 4 explains how measures of technical efficiency can be written as the product of metatechnology ratios and measures of residual technical efficiency; these measures can be viewed as measures of how well technologies are chosen and used. Section 5 presents a number of results concerning hull operators. It then uses these results to establish relationships between technology-specific and metatechnology-specific production possibilities sets. Section 6 (resp. Section 7) explains how free disposal hull (resp. data envelopment analysis) estimators can be used to estimate C and NC technology-specific and metatechnology-specific production possibilities sets. These estimators can also be used to estimate metatechnology ratios and measures of residual technical efficiency. Section 8 discusses related metafrontier models and methods. Section 9 contains an empirical illustration based

[1] Sometimes this Malmquist productivity index, which is most frequently estimated within a frontier-based framework, has been combined with the more traditional meta-production function approach: see, e.g., Fulginiti & Perrin (1998).

[2] Hung, Le Van & Michel (2009) explore the complexities of optimal growth when the union of just two separate C production possibilities sets exhibits a basic nonconvexity.

[3] A Google Scholar search on 11 October 2018 obtained 4420 results for the expression "metafrontier". Furthermore, the key article of O'Donnell et al. (2008) has 736 cites on that same date. Obviously, not all of these articles involved have adopted a convexification strategy and are potentially wrong. Instead of listing culprits, we simply mention a few articles that do not adopt such a convexification strategy: examples include Breustedt, Francksen & Latacz-Lohmann (2007), Huang et al. (2013), Sala-Garrido et al. (2011), Tiedemann, Francksen & Latacz-Lohmann (2011), Verschelde, Dumont, Rayp & Merlevede (2016), among others.

on a secondary data set. The focus here is on the convexification strategy that is common in the mainstream literature. We find evidence that estimates of firm performance are significantly affected by this convexification strategy. Section 10 summarises the article, transposes the key results to alternative estimators and makes a concluding recommendation.

In order to save space and avoid repetition, the remainder of this article is almost totally focused on estimating input distance functions and associated input-oriented measures of firm performance. Extending our work to output distance functions and associated output-oriented measures of performance is a trivial exercise that is left to the reader.

2. Technology and technology-specific frontier

In O'Donnell (2016, p. 328), a *technology* is defined as "a technique, method or system for transforming inputs into outputs.... For most practical intents and purposes, it is convenient to think of a technology as a book of instructions, or recipe". We adopt this definition here and we view a technology as a type of intellectual capital.[4]

Technology can be represented by a *technology-specific production possibilities set* (TPPS). A TPPS is a set containing all input-output combinations that are possible using a given technology. Let $x \in \mathbb{R}_+^M$ and $y \in \mathbb{R}_+^N$ denote vectors of inputs and outputs respectively. Mathematically, the set of all pairs of input and output vectors that can be produced using technology g is

$$t^g = \{(x, y) \in \mathbb{R}_+^{M+N} : x \text{ with technology } g \text{ can produce } y\}. \quad (1)$$

The boundary of this set is referred to as a *technology-specific frontier*. It is common to assume the following:[5]

(T.1) $(x, 0) \in t^g$ for all $x \in \mathbb{R}_+^M$.
(T.2) If $(0, y) \in t^g$, then $y = 0$.
(T.3) t^g is a closed subset of $\mathbb{R}_+^{M+N}$.
(T.4) If $(x, y) \in t^g$ and $(x', -y') \geq (x, -y)$, then $(x', y') \in t^g$.
(T.5) t^g is a C set.
(T.6) If $(x, y) \in t^g$, then there exists $r > 0$ such that $(\lambda x, \lambda^r y) \in t^g$ for all $\lambda > 0$.

These rather traditional assumptions concerning technology g state that: (i) inactivity is possible, (ii) there is no free lunch, (iii) the set of feasible input-output combinations contains all the points on its boundary (closedness), (iv) inputs and outputs are freely (or strongly) disposable, (v) the production possibilities set is convex, and (vi) if inputs are increased by one percent, then outputs can be increased by approximately r percent. The frontier is said to exhibit decreasing returns to scale (DRS), constant returns to scale (CRS) or increasing returns to scale (IRS) as r is less than, equal to, or greater than one (respectively). For more details, see, for example, Hackman (2008). The first and last two assumptions (T.1, T.5 and T.6) are not always maintained in this article.

If assumption T.4 is true, then t^g can be represented using the following technology-specific input distance function:

$$d_I^g(x, y) = \sup_{\lambda \in \mathbb{R}_+} \{\lambda : (x/\lambda, y) \in t^g\}. \quad (2)$$

This function is non-negative, linearly homogeneous in inputs, and no less than unity for all $(x, y) \in t^g$.

3. Metatechnology and metafrontier

We introduce the *technology set* or *metatechnology* Γ as the set of all technologies g (or recipes) that exist in a given period.[6] If we view a technology as a recipe, then we can follow Caselli & Coleman (2006, p. 509) and view a technology set as "a library, containing blueprints, or recipes to turn inputs into outputs". In this article, the set of all input and output vector pairs that are possible using a given technology set Γ (i.e., using some technology that is contained in Γ) is referred to as a *metatechnology-specific production possibilities set* (MTPPS). Mathematically, this set of possible input–output combinations is

$$T^\Gamma = \{(x, y) \in \mathbb{R}_+^{M+N} : \exists g \in \Gamma : x \text{ and } g \text{ can produce } y\}. \quad (3)$$

Equivalently, $T^\Gamma = \cup_{g \in \Gamma} t^g$ where Γ is the technology set. The boundary of a MTPPS is referred to as a *metafrontier* in this article.

Note that the time dimension is not introduced in the above notions and corresponding notations. However, if relevant, this time dimension can be included quite straightforwardly. In this respect, we mention two natural implications. First, technologies can be identified at a given time period t. Then, the technology set Γ contains all technologies g that exist at that given time period. Consequently, all TPPSs and the corresponding MTPPS also depend on that given time period. Second, one technology in particular could be considered at different time periods (e.g., before, during, and after a crisis). Then, interpret this case as having different technologies available, each with their corresponding TPPS. The technology set Γ now contains these "different" technologies that determines the corresponding MTPPS.

If assumption T.4 is true, then T^Γ can be represented using the following metatechnology-specific input distance function:

$$D_I^\Gamma(x, y) = \max_{g \in \Gamma} \{d_I^g(x, y)\}. \quad (4)$$

Equivalently, $D_I^\Gamma(x, y) = \sup_{\lambda \in \mathbb{R}_+} \{\lambda : (x/\lambda, y) \in T^\Gamma\}$. This function is non-negative, linearly homogeneous in inputs, and no less than unity for all $(x, y) \in T^\Gamma$.

4. Technical efficiency

In this article, the input-oriented metatechnology-specific technical efficiency (ITE) of a firm that uses inputs x to produce outputs y using some technology $g \in \Gamma$ is defined as the reciprocal of the metatechnology-specific input distance function:

$$\text{ITE}^\Gamma(x, y) = 1/D_I^\Gamma(x, y). \quad (5)$$

This measure goes back to the coefficient of resource utilisation defined by Debreu (1951, p. 285). It is a radial measure of efficiency that lies in the closed unit interval. It indicates the maximum equiproportionate reduction in x which still allows production of y by some technology $g \in \Gamma$.

If Γ contains more than one technology, then the measure of ITE defined by (5) can be written as the product of an input-oriented metatechnology ratio (IMR) and a measure of residual input-oriented technical efficiency (RITE). Mathematically, the IMR

[4] Other authors use the term "technology" quite differently. For example, Balk (1998, p. 12) uses the term to describe a set of feasible input-output combinations; in this article, sets of feasible input–output combinations are referred to as production possibilities sets. As another example, Bresnahan & Trajtenberg (1995) use the term "general purpose technologies" to refer to goods that can be used for many purposes, e.g., "the steam engine, the electric motor, and semiconductors" (p. 83). In this article, goods are referred to as either inputs or outputs depending on whether they are used in or come out of a given production process.

[5] If only one technology exists, then the "g" notation can be suppressed and T.1 to T.6 collapse to the axioms found in most textbooks.

[6] Other authors use the term "technology set" quite differently. For example, Färe & Primont (1995, p. 8), Coelli, Rao, O'Donnell & Battese (2005, p. 42) and Afsharian & Ahn (2015, p. 6) use the term to describe a set of feasible input-output combinations. O'Donnell (2016) refers to the set of technologies that exist in a given period as a "metatechnology".

relative to the technology set Γ of a firm that uses inputs x and technology g to produce outputs y is

$$IMR^{g\Gamma}(x, y) = d_I^g(x, y)/D_I^\Gamma(x, y). \quad (6)$$

This measure also lies in the closed unit interval. It can be viewed as an input-oriented measure of whether a firm has chosen the best technology that is available. The associated measure of RITE is

$$RITE^g(x, y) = 1/d_I^g(x, y). \quad (7)$$

This measure also lies in the closed unit interval. It indicates the maximum equiproportionate reduction in x which still allows production of y when using technology g. It can also be viewed as the component of ITE that remains after accounting for the IMR (hence the term "residual"). Eqs. (4), (5) and (7) imply that

$$ITE^\Gamma(x, y) = \min_{g \in \Gamma}\{RITE^g(x, y)\}. \quad (8)$$

An analogous definition of output-oriented technical efficiency (OTE) based on a similar enumeration over groups has recently been defined by Afsharian & Ahn (2015). Note that some of the components in (8) can be undefined for some input-output combinations that are not contained in the group technology composing the technology or metatechnology (see Briec & Kerstens, 2009 for details on infeasibilities). Finally, Eqs. (5)–(7) imply that

$$ITE^\Gamma(x, y) = IMR^{g\Gamma}(x, y) \cdot RITE^g(x, y). \quad (9)$$

Thus, technical efficiency can be decomposed into the product of a metatechnology ratio (measuring how close a technology-specific frontier is to the metafrontier) and a measure of residual technical efficiency (measuring how close a firm is operating to the technology-specific frontier). Eq. (9) is an input-oriented version of the output-oriented efficiency decomposition in O'Donnell et al. (2008, Eq. 10).

5. Different hull operators and their properties

To inform our discussion of convexification, we first establish some results concerning the monotonic hull, conical hull, and C hull of a general set $A \subseteq \mathbb{R}_+^{M+N}$.

Definition 5.1. For the set $A \subseteq \mathbb{R}_+^{M+N}$,

(a) the monotonic hull of A is given by $mon(A) = \{z \in \mathbb{R}_+^{M+N} \mid \exists u \in A, \exists (x_+, y_-) \in \mathbb{R}_+^M \times \mathbb{R}_-^N$ such that $z = u + (x_+, y_-)\}$;
(b) the conical hull of A is given by $cone(A) = \{z \in \mathbb{R}_+^{M+N} \mid \exists u \in A, \exists \lambda \in \mathbb{R}_{++}$ such that $z = \lambda u\}$;
(c) the C hull of A is given by $conv(A) = \{z \in \mathbb{R}_+^{M+N} \mid \exists u_i \in A, \exists \alpha_i \in \mathbb{R}_+$ with $\sum_i \alpha_i = 1$ such that $z = \sum_i \alpha_i u_i\}$.

Obviously, the different hull operators of Definition 5.1 can be combined in several ways. To mention just a few possibilities for $A \subseteq \mathbb{R}_+^{M+N}$, the C conical hull of A is $conv(cone(A))$, the C monotonic hull of A is $conv(mon(A))$, the monotonic C hull of A is $mon(conv(A))$, and the conical C hull of A is $cone(conv(A))$. The following proposition says that the order in which these hull operators are applied on a given set does not matter.

Proposition 5.1. *For the set $A \subseteq \mathbb{R}_+^{M+N}$,*

(a) $mon(conv(A)) = conv(mon(A))$;
(b) $cone(conv(A)) = conv(cone(A))$;
(c) $mon(cone(A)) = cone(mon(A))$.

The proof of this proposition and all other propositions is found in Appendix 1.

Fig. 1 illustrates the hull operators of Definition 5.1 on a set A containing 32 data points in $\mathbb{R}_+^2$. The C hull $conv(A)$ is the region enclosed by the dashed polyline starting from observation 1 and continuing all the way round to observation 1 again. The monotonic hull $mon(A)$ consists of the region restricted to the first quadrant (i.e., the region with positive x- and y-coordinates) located below and to the right of the solid polyline starting vertical at the bottom towards observation 3 and then continuing to observations 2, 1, 23, 15, 31 and 16 using horizontal and vertical connections to end horizontally from observation 16 onwards. Unifying both these regions results in the C monotonic hull $conv(mon(A))$. This is the region restricted to the first quadrant below and to the right of the polyline starting with the vertical solid line to observation 2, then continuing via the dashed lines to observations 1, 15 and 16 to end with the horizontal solid line from observations 16 onwards. The C conical hull $conv(cone(A))$ consists of the region enclosed between the two dotted lines.

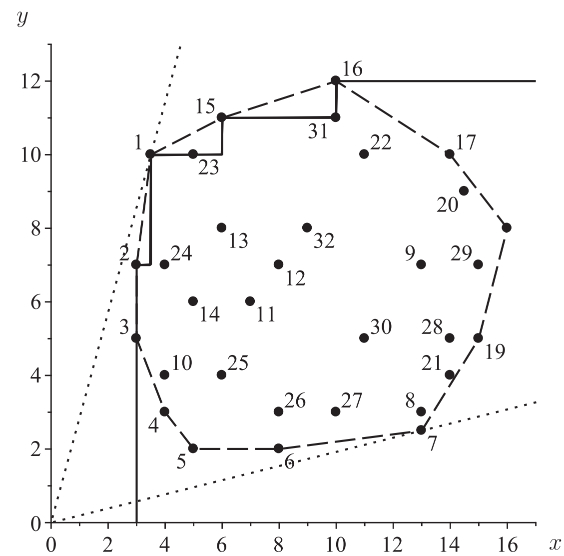

Fig. 1. Different Hull operators Applied to the Same Set.

The next proposition shows that a given set is contained in all hulls and that all hull operators preserve subset relationships.

Proposition 5.2. *For the sets $A, B \subseteq \mathbb{R}_+^{M+N}$ with $A \subseteq B$,*

(a) $A \subseteq mon(A)$, $A \subseteq cone(A)$, *and* $A \subseteq conv(A)$;
(b) $mon(A) \subseteq mon(B)$, $cone(A) \subseteq cone(B)$, *and* $conv(A) \subseteq conv(B)$.

In combination with the union operator on sets, we can establish the following results (to the best of our knowledge, these results are new):[7]

Proposition 5.3. *For the sets $A, B \subseteq \mathbb{R}_+^{M+N}$,*

(a) $mon(A) \cup mon(B) = mon(A \cup B)$;
(b) $cone(A) \cup cone(B) = cone(A \cup B)$;
(c) $conv(A) \cup conv(B) \subseteq conv(A \cup B)$.

Thus, the union of the monotonic (resp. conical) hulls of two sets is equal to the monotonic (resp. conical) hull of the union of these two sets. However, the union of the C hulls of two sets is only a subset of the C hull of the union of these two sets. The following result can now be stated.

Proposition 5.4. *For the sets $A, B \subseteq \mathbb{R}_+^{M+N}$,*

(a) $conv(mon(A)) \cup conv(mon(B)) \subseteq conv(mon(A \cup B))$;
(b) $conv(cone(A)) \cup conv(cone(B)) \subseteq conv(cone(A \cup B))$;
(c) $cone(mon(A)) \cup cone(mon(B)) = cone(mon(A \cup B))$.

[7] Operations on sets (or composition rules) have to our knowledge been discussed theoretically only while maintaining convexity: see, e.g., Ruys (1974).

Again, only the convexity operator yields subset relationships. Note that converse properties of (a) and (b) do not hold true. We leave it to the reader to illustrate this using a counterexample. The counterexample used in the proof of Proposition 5.3(c) can serve as inspiration.

All of the aforementioned hull operators and their properties can be applied to the production possibilities sets introduced in Sections 2 and 3. Propositions 5.3 and 5.4 imply:

Proposition 5.5. *If $T^\Gamma = \cup_{g\in\Gamma} t^g$ for the technology set Γ, then*

(a) $\cup_{g\in\Gamma} conv(mon(t^g)) \subseteq conv(mon(T^\Gamma))$;
(b) $\cup_{g\in\Gamma} conv(cone(t^g))) \subseteq conv(cone(T^\Gamma))$;
(c) $\cup_{g\in\Gamma} conv(cone(mon(t^g)))) \subseteq conv(cone(mon(T^\Gamma)))$;
(d) $\cup_{g\in\Gamma} mon(t^g) = mon(T^\Gamma)$;
(e) $\cup_{g\in\Gamma} cone(mon(t^g)) = cone(mon(T^\Gamma))$.

Proposition 5.5(a) says that the C monotonic hull of the MTPPS contains the union of all C monotonic hulls of each TPPS. Equality is only obtained in restrictive special cases. Proposition 5.5(b) says that the C conical hull of the MTPPS contains the union of all C conical hulls of each TPPS. Proposition 5.5(c) says that the C conical monotonic hull of the MTPPS contains the union of all C conical monotonic hulls of each TPPS. Proposition 5.5(d) says that the monotonic hull of the MTPPS equals the union of all monotonic hulls of each TPPS. Proposition 5.5(e) says that the conical monotonic hull of the MTPPS equals the union of all conical monotonic hulls of each TPPS.

The convexification strategy that is found in the mainstream efficiency literature is equivalent to assuming explicitly or implicitly that the subset relationships in Proposition 5.5(a)–(c) can be replaced for practical purposes by an equality relationship. To the best of our knowledge, the consequences of such a convexification strategy have not been thoroughly assessed in the literature.

In the next two sections, we consider the consequences of this convexification strategy for nonparametric estimators of technology-specific frontiers and associated metafrontiers. These estimators are all associated to initial observations. Therefore, we introduce the following notations. Assume that n observations $(x_1, y_1), \ldots, (x_n, y_n)$ are available, and that technology g is determined by $n^g \leq n$ of these observations. To identify these particular observations, consider the one-to-one index function ϕ_g mapping the set $\{1, \ldots, n^g\}$ into the set $\{1, \ldots, n\}$. Then, $(x_{\phi_g(i)}, y_{\phi_g(i)})$ denotes the ith generating observation of g. To illustrate these notations, consider the case where technology g is determined by the three observations (x_1, y_1), (x_3, y_3) and (x_8, y_8). Then, $n^g = 3$ and $\phi_g : \{1, 2, 3\} \to \{1, \ldots, n\}$ with $\phi_g(1) = 1$, $\phi_g(2) = 3$ and $\phi_g(3) = 8$.

6. Free disposal hull estimators

If every TPPS is NC and the associated technology exhibits variable returns to scale (VRS), then an asymptotically unbiased estimator for this TPPS is $t^g_{NC,VRS} = mon(s^g)$ with $s^g = \{(x_{\phi_g(i)}, y_{\phi_g(i)}) : i = 1, \ldots, n^g\}$ the set of n^g initial observations determining technology g. Equivalently,

$$t^g_{NC,VRS} = \Bigg\{ (x, y) \in \mathbb{R}^{M+N}_+ : \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \{0, 1\} \Bigg\}. \quad (10)$$

Proposition 5.5(d) implies that the associated asymptotically unbiased estimator for T^Γ is $T^\Gamma_{NC,VRS} = mon(\cup_{g\in\Gamma} s^g)$. Equivalently,

$$T^\Gamma_{NC,VRS} = \Bigg\{ (x, y) \in \mathbb{R}^{M+N}_+ : \sum_{g\in\Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{g\in\Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{g\in\Gamma} \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \{0, 1\} \Bigg\}. \quad (11)$$

Fig. 2(a) illustrates the relationship between the estimators (10) and (11) for the single-input-single-output case when only two technologies exist. In this figure, the estimated TPPS $t^1_{NC,VRS}$ (resp. $t^2_{NC,VRS}$) consists of all points between the polyline $A_1B_1C_1D_1E_1F_1G_1H_1I_1$ (resp. $A_2B_2C_2D_2E_2F_2G_2H_2I_2$) and the horizontal axis. The estimated MTPPS $T^{\{1,2\}}_{NC,VRS}$ consists of all points between the polyline $A_1B_1C_1D_1PB_2QF_1RD_2E_2F_2G_2H_2I_2$ and the horizontal axis. Notice the equality relationship of Proposition 5.5(d).

If every TPPS is NC and the associated technology exhibits constant returns to scale (CRS), then an asymptotically unbiased estimator for this TPPS is $t^g_{NC,CRS} = cone(mon(s^g))$ with $s^g = \{(x_{\phi_g(i)}, y_{\phi_g(i)}) : i = 1, \ldots, n^g\}$ the set of n^g initial observations determining technology g. Equivalently,

$$t^g_{NC,CRS} = \Bigg\{ (x, y) \in \mathbb{R}^{M+N}_+ : \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \lambda_{\phi_g(i)} \in \{0, 1\}, \delta \geq 0 \Bigg\}. \quad (12)$$

Proposition 5.5(e) implies that the associated asymptotically unbiased estimator for T^Γ is $T^\Gamma_{NC,VCS} = cone(mon(\cup_{g\in\Gamma} s^g))$. Equivalently,

$$T^\Gamma_{NC,CRS} = \Bigg\{ (x, y) \in \mathbb{R}^{M+N}_+ : \sum_{g\in\Gamma} \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{g\in\Gamma} \sum_{i=1}^{n^g} \delta\lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \lambda_{\phi_g(i)} \in \{0, 1\}, \delta \geq 0 \Bigg\}. \quad (13)$$

The estimators (10)–(13) are commonly known as free disposal hull (FDH) estimators. Single output FDH estimators go back to Afriat (1972). If the technology-specific frontiers exhibit VRS (resp. CRS), then the estimator (10) (resp. (12)) can be used to construct an asymptotically unbiased estimator for the measure of RITE (7). The associated estimators (11) and (13) can be used to construct asymptotically unbiased estimators for the measure of ITE (5). FDH estimation of either a TPPS or MTPPS requires solving a mixed integer (non)linear program for each evaluated observation. However, Leleu (2006) and Briec, Kerstens & Vanden Eeckaut (2004) propose a linear programming (LP) solution and a closed form solution based on an implicit enumeration strategy, respectively.

7. Data envelopment analysis estimators

If every TPPS is C and the associated technology exhibits VRS, then an asymptotically unbiased estimator for this TPPS is $t^g_{C,VRS} = conv(mon(s^g)) = mon(conv(s^g))$ with $s^g = \{(x_{\phi_g(i)}, y_{\phi_g(i)}) : i = 1, \ldots, n^g\}$ the set of n^g initial observations determining technology g. Equivalently,

$$t^g_{C,VRS} = \Bigg\{ (x, y) \in \mathbb{R}^{M+N}_+ : \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \leq x, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \geq y, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \mathbb{R}_+ \Bigg\}. \quad (14)$$

This estimator is the convexified version of (10). It differs from (10) in that the nonnegative activity (or intensity) variables $(\lambda_{\phi_g(i)})$

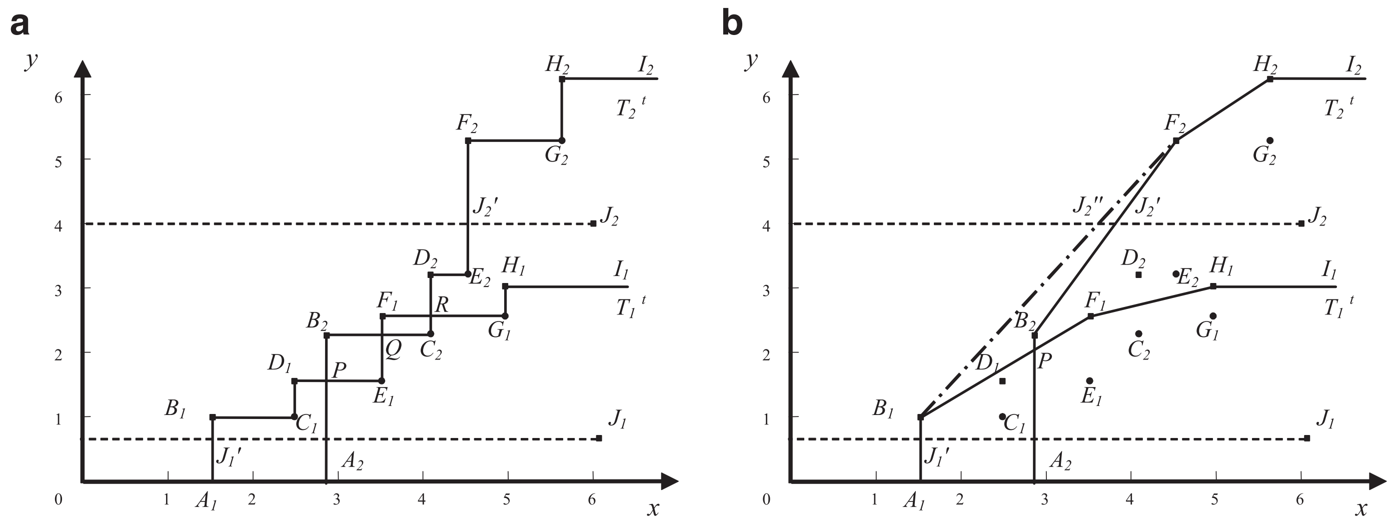

Fig. 2. (a) NC and (b) C Group-Specific Technologies and NC Metatechnologies.

are no longer restricted to be binary integers. The associated asymptotically unbiased estimator for T^Γ is

$$T^\Gamma_{C,VRS} = \cup_{g\in\Gamma} conv(mon(s^g)). \quad (15)$$

This estimator goes back at least as far as Tiedemann et al. (2011, p. 578). Proposition 5.5(a) implies that it is not necessarily equal to the convexified version of (11). The convexified version of (11) is $H^\Gamma_{C,VRS} = conv(mon(\cup_{g\in\Gamma} s^g))$. Equivalently,

$$H^\Gamma_{C,VRS} = \left\{ (x, y) \in \mathbb{R}^{M+N}_+ : \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \le x, \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \ge y, \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \mathbb{R}_+ \right\}. \quad (16)$$

O'Donnell et al. (2008, p. 238) use an estimator of this type[8] to construct an estimator of OTE. Proposition 5.5(a) implies that $H^\Gamma_{C,VRS} \supseteq T^\Gamma_{C,VRS}$. Thus, except in restrictive special cases (e.g., only one technology exists), it is an asymptotically biased estimator of the MTPPS. The basic question we address in this article is whether it yields efficiency estimates that are close to the estimates obtained using the asymptotically unbiased estimator (15).

To the best of our knowledge, no single study has ever documented this bias issue. Tiedemann et al. (2011) only compare C TPPSs to the correct NC MTPPS defined as the union of these TPPSs, but they ignore the bias issue (see also Sala-Garrido et al., 2011). This bias issue is only partly documented in Huang et al. (2013) and in the unpublished paper of Breustedt et al. (2007): the former article lists the units whose efficiency measure is different on the true NC compared to the biased convexified MTPPSs (see their Table 4), the latter study illustrates these same differences in metafrontier efficiency measures mainly graphically. But, none of these studies reports any test statistic regarding these differences in metafrontier efficiency measures.

Fig. 2(b) illustrates the relationship between the estimators (14)–(16) for the single-input-single-output case when only two technologies exist.[9] In this figure, the estimated TPPS $t^1_{C,VRS}$ (resp. $t^2_{C,VRS}$) consists of all points between the horizontal axis and the polyline $A_1B_1F_1H_1I_1$ (resp. $A_2B_2F_2H_2I_2$). The estimated MTPPS $T^{\{1,2\}}_{C,VRS}$ consists of all points between the horizontal axis and the polyline $A_1B_1PB_2F_2H_2I_2$. While each estimated TPPS is convex, the estimated MTPPS is clearly non-convex. The estimator (16) convexifies this NC set by adding the region $B_1PB_2F_2B_1$; the resulting set $H^{\{1,2\}}_{C,VRS}$ consists of all points between the horizontal axis and the polyline $A_1B_1F_2H_2I_2$. If we were to add an additional technology, then the set $T^\Gamma_{C,VRS}$ may still not equal the C set $H^\Gamma_{C,VRS}$; an additional TPPS might fill up part of the region determined by $B_1PB_2F_2B_1$, but it could easily create further non-convexities to the left or to the right of the existing one. Notice in Fig. 2(b) the subset relationship in Proposition 5.5(a). Obviously, one must realize that in a multiple-input-multiple-output setting (rather than in a single-input-single-output setting), the union of more than two TPPSs is only by sheer coincidence going to end up yielding a C set. The most likely outcome is simply that the resulting MTPPS is non-convex.

If every TPPS is C and the associated technology exhibits CRS, then an asymptotically unbiased estimator for this TPPS is $t^g_{C,CRS} = conv(cone(mon(s_g)))$ with $s^g = \{(x_{\phi_g(i)}, y_{\phi_g(i)}) : i = 1, \ldots, n^g\}$ the set of n^g initial observations determining technology g. Equivalently,

$$t^g_{C,CRS} = \left\{ (x, y) \in \mathbb{R}^{M+N}_+ : \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \le x, \sum_{i=1}^{n^g} \lambda_i y_{\phi_g(i)} \ge y, \lambda_{\phi_g(i)} \in \mathbb{R}_+ \right\}. \quad (17)$$

This estimator is the convexified version of (12). Again, it differs from (12) in that the nonnegative activity variables are no longer restricted to be binary integers. The associated asymptotically unbiased estimator for T^Γ is

$$T^\Gamma_{C,CRS} = \cup_{g\in\Gamma} conv(cone(mon(s^g))). \quad (18)$$

Proposition 5.5(c) implies that this is not necessarily equal to the convexified version of (13). The convexified version of (13) is $H^\Gamma_{C,CRS} = conv(cone(mon(\cup_{g\in\Gamma} s^g)))$. Equivalently,

$$H^\Gamma_{C,CRS} = \left\{ (x, y) \in \mathbb{R}^{M+N}_+ : \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)} \le x, \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)} \ge y, \lambda_{\phi_g(i)} \in \mathbb{R}_+ \right\}. \quad (19)$$

[8] O'Donnell et al. (2008) effectively estimate OTE under the assumption that there is no technical change.

[9] The data points in this figure are, in fact, the data points that were depicted earlier in Fig. 2. However, each TPPS is now assumed to be convex.

Proposition 5.5(c) implies that $H^{\Gamma}_{C,CRS} \supseteq T^{\Gamma}_{C,CRS}$. Thus, once again, except in restrictive special cases, it is an asymptotically biased estimator of the MTPPS. Again, the basic question is whether it yields efficiency estimates that are close to the estimates obtained using the asymptotically unbiased estimator (18).

The estimators (14), (16), (17) and (19) are commonly known as data envelopment analysis (DEA) estimators. Single output DEA estimators also go back to at least Afriat (1972). Multi-output DEA estimators appear to have been introduced to the literature by Banker, Charnes & Cooper (1984) and Färe, Grosskopf & Lovell (1983). If the TPPS exhibit VRS (resp. CRS), then the estimator (14) (resp. (17)) can be used to construct an asymptotically unbiased estimator for the measure of RITE (7). This requires solving an LP problem for each evaluated observation (see Hackman (2008) or Ray (2004)). The associated estimators (15) and (18) can be used to construct asymptotically unbiased estimators for the measure of ITE (5). Recently, Afsharian & Podinovski (2018) demonstrate that this can be achieved in both cases by solving a single LP problem. For the reasons given earlier, the estimators (16) and (19) cannot generally be used to construct an asymptotically unbiased estimator for the measure of ITE.

8. Related metafrontier models and methods

This article has focused on metafrontier methods for estimating the gaps between TPPSs and the MTPPS. Metafrontier methods can also be used to estimate the gaps between other types of production possibilities sets. For example, they can be used to estimate the gaps between period-and-state-contingent production possibilities sets and period-specific production possibilities sets. A period-and-state-contingent production possibilities set is a set containing all input-output combinations that are possible in a given period in a given state of the production environment (or state of nature). Suppose there are $S \in \mathbb{Z}_+$ possible states of nature. Mathematically, the set of all input and output vector pairs that are possible in period t in state s is $T^t(s) = \{(x, y) \in \mathbb{R}^{M+N}_+ :$ x can produce y in period t in state $s\}$. The boundary of this set is a period-and-state-contingent frontier. The associated period-specific production possibilities set is $T^t = \cup_{s \in \Omega} T^t(s)$ where $\Omega = \{1, \dots, S\}$. If inputs are freely disposable, then $T^t(s)$ can be represented using the following period-and-state-contingent input distance function:

$$D^t_I(x, y, s) = \sup_{\lambda \in \mathbb{R}_+} \left\{ \lambda : (x/\lambda, y) \in T^t(s) \right\}. \tag{20}$$

This distance function gives the largest factor by which it is possible to reduce x and still produce y in period t in state s. The associated period-specific input distance function is

$$D^t_I(x, y) = \max_{s \in \Omega} \{ D^t_I(x, y, s) \}. \tag{21}$$

This distance function gives the largest factor by which it is possible to reduce x and still produce y in period t. An associated input-oriented measure of firm and environmental performance is the following input-oriented technical efficiency and environmental effect (ITEEE):

$$ITEEE^t(x, y) = 1/D^t_I(x, y). \tag{22}$$

This is a radial measure of performance that lies in the closed unit interval. If there is only one state of nature (i.e., there are no environmental effects), then it is equivalent to the measure of ITE defined by (5). If $S > 1$, then it can be written as the product of an input-oriented environmental effect (IEE) and a measure of ITE. Mathematically, the IEE a firm that uses inputs x to produces outputs y in period t in state s is

$$IEE^t(x, y, s) = D^t_I(x, y, s)/D^t_I(x, y). \tag{23}$$

This measure also lies in the closed unit interval. It can be viewed as an input-oriented measure of whether a firm is operating in the best production environment. The ITE of the firm is

$$ITE^t(x, y, s) = 1/D^t_I(x, y, s). \tag{24}$$

This measure also lies in the closed unit interval. It can still be viewed as a measure of how well the firm makes use of the technologies that exist in period t. Eqs. (21), (22) and (24) imply that

$$ITEEE^t(x, y) = \min_{s \in \Omega} \{ ITE^t(x, y, s) \}. \tag{25}$$

Finally, Eqs. (22)–(24) imply that

$$ITEEE^t(x, y) = ITE^t(x, y, s) \cdot IEE^t(x, y, s). \tag{26}$$

Eqs. (22)–(26) are analogous to Eqs. (5)–(9). Associated FDH and DEA estimators of these quantities are analogous to the estimators discussed in Sections 6 and 7.

The message to take from this discussion is that various functions and measures of performance associated with different production environments have the same mathematical structure, but not necessarily the same interpretation, as those associated with the selection and use of technologies. From a mathematical viewpoint, the distinction between technologies and environments is immaterial. The distinguishing feature of metafrontier models is that firms can be classified into groups. If the classification is not obvious, then there are several multivariate statistical methods that can be used to classify the units in a sample into either some natural groups (e.g., using some variant of cluster analysis) or a set of existing groups (e.g., using discriminant analysis and its variations). Examples include Samoilenko & Osei-Bryson (2010) and Llorca, Orea & Pollitt (2014), among others. Along similar lines, frontier estimates have been used in a variety of ways to distinguish strategic groups (e.g., Athanassopoulos (2003), Warning (2004), among others). As long as groups can be identified, the metafrontier framework is applicable.

The short article of Pastor & Lovell (2005) proposes a global Malmquist productivity index based upon the specification of a single global technology constructed by taking a convex cone over all data for all observations and all time periods simultaneously. It has the key attractive feature of being circular (just like any fixed base index). However, Afsharian & Ahn (2015) analyse this proposal in detail and fundamentally criticise the Pastor & Lovell (2005) proposal for adopting – in our parlance – a convexification strategy that is unwarranted. In particular, these authors point out that this strategy (1) neglects the role of each contemporaneous technology in the determination of the global benchmark technology; (2) assumes that convex combinations of observations across time periods are feasible; and (3) previously computed results by the global Malmquist index can change significantly when a new time period is incorporated. Therefore, Afsharian & Ahn (2015) plea to use a proper union of convex cones defined per contemporaneous technology (instead of the convex cone over the union of all observations).

9. Empirical illustration: hydroelectric power plants

In this section, we illustrate the implications of convexification using data that have previously been used by Atkinson & Dorfman (2009) to assess the performance of Chilean hydroelectric power plants.

There are two main techniques (or technologies) used to generate hydroelectric power in Chile. The first technology (given index 1) involves building a large dam on a river to store water. Water is then released from the dam to spin turbines that generate electricity. The advantage of so-called dam systems is that electricity generation is de-coupled from river flows. The second type

Table 1
C-NC and NC-NC Estimates of ITE, RITE and the IMR.

		C-NC			NC–NC			Inf.
		ITE	RITE	IMR	ITE	RITE	IMR	
All 192	#Eff. Obs.	21	24	163	154	168	175	
Observations	Geom. Mean	0.7582	0.8046	0.9424	0.9391	0.9629	0.9752	
	Stand. Dev.	0.2009	0.1791	0.1260	0.1378	0.1076	0.0878	
	Min.	0.1325	0.2094	0.3740	0.3154	0.3154	0.3866	
Li-test†		34.15	37.15	2.13				
p-value		(0.000)	(0.000)	(0.022)				
60 Dam	#Eff. Obs.	1	4	31	39	52	44	
Observations	Geom. Mean	0.6751	0.8163	0.8271	0.8811	0.9547	0.9229	29
	Stand. Dev.	0.2141	0.1645	0.1896	0.1867	0.1181	0.1495	
	Min.	0.1325	0.2681	0.3740	0.3212	0.3212	0.3866	
132 ROR	#Eff. Obs.	20	20	132	115	116	131	
Observations	Geom. Mean	0.7994	0.7994	1.0000	0.9667	0.9667	$\approx$1.000	0
	Stand. Dev.	0.1853	0.1853	0.0000	0.1024	0.1024	0.0004	
	Min.	0.2094	0.2094	1.0000	0.3154	0.3154	0.9953	

† Li test: exact p values are reported in round brackets underneath.

of hydroelectric power technology (given index 2) involves merely diverting river flows through turbines. The advantage of so-called run-of-river (ROR), or diversion, systems is that they are relatively inexpensive and have relatively little impact on the environment. A disadvantage of these systems is that they cannot be used to match electricity generation with consumer demand.[10]

By construction, the technology set $\Gamma = \{1, 2\}$. The sample comprises data on $M = 3$ inputs and $N = 1$ output for 16 Chilean hydroelectric power plants over the 12 months of the year 1997. There are 5 dam plants and 11 ROR plants in our sample. The data are publicly available on the web site of the *Journal of Applied Econometrics*.[11] The single output is electricity generated (in gigawatt hours). The three inputs are labor (thousands of workers), capital (real pesos), and water (cubic meters). More details concerning the data can be accessed from Atkinson & Dorfman (2009) and Atkinson & Halabí (2005).

Our knowledge of hydroelectric power generation in Chile leads us to believe that it *may* be possible for the manager of a given (dam or ROR) plant to use a given input vector to produce a given level of output for some of the time, and then use a different input vector to produce a different level of output the rest of the time. This suggests that each TPPS *may* be convex. Consequently, we began by estimating these TPPSs t^1 and t^2 using the "convexifying" DEA estimator (14). It is also our understanding that, given the different types of capital involved in constructing different plants, the manager of a given plant cannot operate a dam system for some of the time and then operate an ROR system the rest of the time. This suggests that the MTPPS should not be convexified. Consequently, we estimated the MTPPS using the DEA estimator (15). Descriptive statistics for the associated estimates of ITE, RITE and IMR are reported in the columns labelled C-NC in Table 1 (the acronym C-NC indicates that the TPPSs have been convexified, but the MTPPS has not been convexified). Of course, TPPSs may *not* be convex. Consequently, we also estimated the TPPSs using the non-convexifying FDH estimator (10). The associated estimator of the period-specific production possibilities set is the non-convexifying FDH estimator (11). Descriptive statistics for the associated estimates of ITE, RITE and IMR are reported in the columns labelled NC–NC in Table 1 (the acronym NC–NC indicates that neither the TPPSs nor the MTPPS have been convexified). In Table 1, both C-NC and NC–NC results are reported in two blocks of three columns each. The last column contains the number of infeasible solutions.[12]

Explaining the rows in Table 1, the first block of numbers in Table 1 contains summary statistics for all $16 \times 12 = 192$ observations in the sample. The next two blocks of numbers contain summary statistics for the $n^1 = 5 \times 12 = 60$ dam observations and the $n^2 = 11 \times 12 = 132$ ROR observations. The first row in each block reports the number of efficient observations (i.e., the number of times the relevant measure of performance is estimated to be 1). The next three rows in each block report the geometric averages,[13] standard deviations, and minima of the relevant estimates.

Four observations can be made with regard to Table 1. First, by construction, estimates of ITE obtained using the estimators (14) and (15) can be no higher than the estimates obtained using the estimators (10) and (11) (i.e., estimates of ITE obtained using the C-NC model can be no higher than those obtained using the NC–NC model). Table 1 reveals that, in the case of Chilean hydroelectric power generators, estimates of ITE obtained using (14) and (15) are on average $1 - 0.7582/0.9391 = 19.3\%$ lower than estimates obtained using (10) and (11).

Second, also by construction, estimates of RITE obtained using the estimator (14) can be no higher than the estimates obtained using the estimator (10) (i.e., estimates of RITE obtained using the C-NC model can also be no higher than those obtained using the NC–NC model). Table 1 reveals that, in our application, estimates of RITE obtained using (14) are on average $1 - 0.8046/0.9629 = 16.4\%$ lower than estimates obtained using (10).

Third, in theory, estimates of the IMRs obtained using the estimators (14) and (15) can be either higher or lower than the estimates obtained using the estimators (10) and (11) (i.e., estimates of the IMRs obtained using the C-NC model can be either higher or lower than those obtained using the NC–NC model). Table 1 reveals that, in our application, estimates of the IMRs obtained using (14) and (15) are on average $1 - 0.9424/0.9752 = 3.4\%$ lower than estimates obtained using (10) and (11). The minima reported for the ROR observations reveal that there was at least one observation where the IMR obtained using the estimators (14) and (15) was higher than the IMR obtained using the estimators (10) and (11).

Finally, the IMR estimates obtained using the C-NC (resp. NC–NC) model and the 132 ROR observations are all (resp. nearly all) equal to one. This indicates that ROR hydroelectric power systems

[10] A third hydroelectric power technology, called pumped storage, can be used to match electricity generation with daily or weekly consumer demand. However, there are no pumped storage plants in our dataset.

[11] See: http://qed.econ.queensu.ca/jae/.

[12] See Briec & Kerstens (2009) for an extensive discussion on the occurrence of infeasibilities for general distance functions.

[13] The use of geometric averages guarantees that the multiplicative decomposition (9) holds exactly.

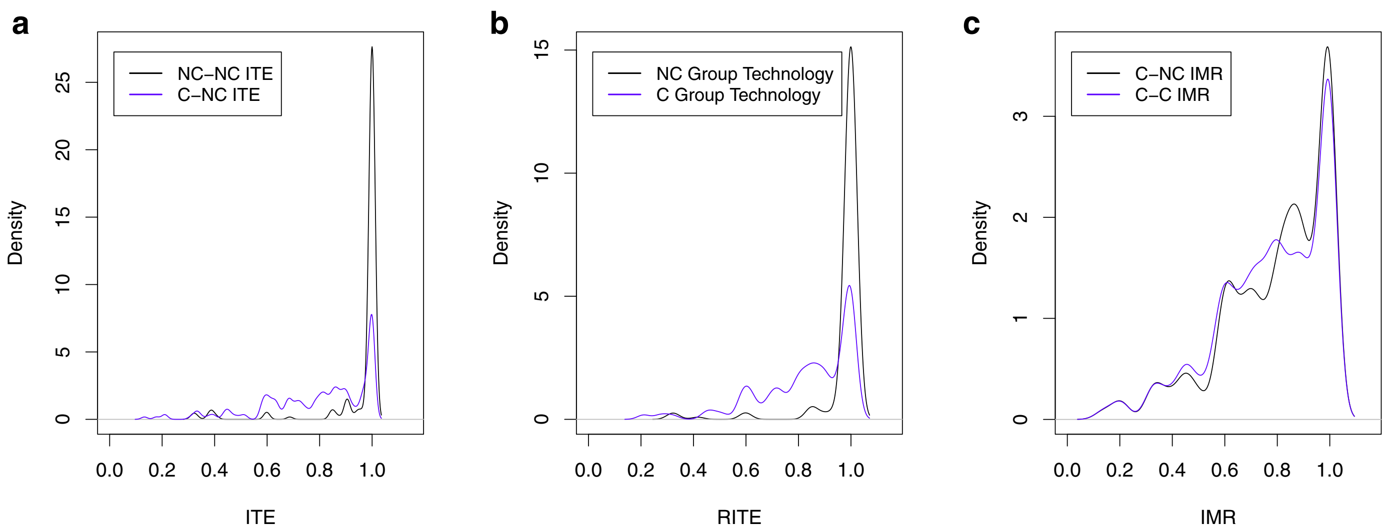

Fig. 3. Kernel density estimates of (a) ITE, (b) RITE, and (c) IMRs.

are superior to dam systems. Indeed, we estimate that approximately half of the observed input-output combinations of ROR plants would not even have been feasible using dam plants (i.e., they lie outside the estimated dam-specific production possibilities set). As far as we know, there are only a handful of other studies that use metafrontier methods to determine the inferiority or superiority of different technologies. For example, Sala-Garrido et al. (2011) compare four wastewater treatment technologies and find that one technology dominates all others.

Fourth, among dam plants there are 29 instances of infeasible solutions when computing some of the distances to the C TPPSs in determining ITE. This amounts to about 15% of the sample. By contrast, ROR plants do not encounter any infeasible solutions at all.

Fig. 3(a) displays the kernel densities of the ITE estimates that were summarised at the top of Table 1.[14] The two densities appear to be quite different. For a formal assessment of this difference, we employ a nonparametric test initially proposed by Li (1996). This test has been refined by Fan & Ullah (1999) and others: the most recent development is by Li, Maasoumi & Racine (2009). This nonparametric test analyzes the differences between entire distributions instead of focusing on, for instance, first moments (as, e.g., the Wilcoxon signed-ranks test). It tests the statistical significance of differences between two kernel-based estimates of density functions, f and g, of a random variable x. The null hypothesis states the equality of both density functions almost everywhere ($H_0 : f(x) = g(x)$ for all x). The alternative hypothesis negates the equality of both density functions (H_1: $f(x) \neq g(x)$ for some x). This test is valid for both dependent and independent variables: observe that dependency is a characteristic of frontier estimators (i.e., efficiency levels depend on sample size, among others). Simar & Zelenyuk (2006) fine tune this test statistic further for nonparametric frontier estimators to circumvent the problem of spurious mass at the boundary: their Algorithm I ignores the boundary estimates and their Algorithm II smooths boundary estimates by adding uniform noise of an order of magnitude less than the order of magnitude of noise added by the specific estimator. Monte Carlo evidence suggests that Algorithm II performs slightly better overall, though the power of the test statistic decreases with the dimensionality of the production specification.[15] In short, we adopt the Li et al. (2009) version of this test amended with Algorithm II from Simar & Zelenyuk (2006). For this test statistic, we report on the next line the exact p value using 2000 bootstrap replications starting from a conventional 5% significance level (i.e., $\alpha = 0.05$). The test statistic was calculated to be 34.15. The p value indicates that we can reject the null hypothesis and conclude that the two distributions are significantly different.

Fig. 3(b) displays the kernel densities of the RITE estimates that were summarised at the top of Table 1. Again, the two densities seem quite different. In this case, the Li test statistic amounts to 37.15. The p value indicates that we can again reject the null hypothesis and conclude that the two distributions are significantly different.

Fig. 3(c) displays the kernel densities of the IMRs that were summarised at the top of Table 1. In this case, the two densities appear to be quite similar. The Li test statistic was calculated to be only 2.13. The large p value indicates that we do not reject the null hypothesis that the two distributions are equal.

The estimates reported in Table 1 and Fig. 3(a)–(c) are perfectly consistent with what we understand about the different types of physical capital used in Chilean hydroelectric power generation. Engineering considerations have led us to understand that it is not possible for the manager of a given (dam or ROR) plant to produce electricity using a linear combination of the physical capital used in a dam system and the physical capital used in an ROR system. This implies that it is not appropriate to follow common practice and convexify the MTPPS. To assess the impact of (inappropriately) convexifying this set, we estimated the MTPPS using the convexifying DEA estimator (16). Descriptive statistics for the associated estimates of ITE and IMR are reported in the columns labelled C–C in Table 2 (the acronym C–C indicates that the TPPSs and MTPPS have all been convexified).[16]

Most of the column and row labels in Table 2 are self explanatory. The columns labelled “Difference” report the differences between the C-NC and the C–C results. For example, the average

[14] Each density was estimated using $N = 192$ observations. For reasons of comparability, a common Sheather and Jones plug-in bandwidth was used (see, e.g., Sheather (2004)).

[15] In fact, there is not much theoretical argument to adopt either algorithm. Simar & Zelenyuk (2006, p. 508) concede: “Although these ways of curing the discontinuity problem by smoothing or deleting the efficiency scores equal to unity is somewhat ad hoc, at this point they seem to be the only two approaches that work well.”

[16] Descriptive statistics for the associated RITE estimates were already reported in the C-NC column in Table 1.

Table 2
C-NC and C–C estimates of ITE and the IMR.

		ITE			IMR		
		C-NC	C–C	Difference	C-NC	C–C	Difference
All 192	# Eff. Obs.	21	23	88	163	71	88
Observations	Arith. Mean.	0.7940	0.7770	0.0170	0.9541	0.9335	0.0206
	Stand. Dev.	0.2009	0.2030	0.0470	0.1260	0.1317	0.0554
	Min.	0.1325	0.1325	0.0000	0.3740	0.3740	0.0000
Li-test†				−1.32			15.21
p-value				(0.951)			(0.000)
60 Dam	# Eff. Obs.	1	2	18	31	1	18
Observations	Arith. Mean.	0.7205	0.6820	0.0385	0.8530	0.8080	0.0450
	Stand. Dev.	0.2141	0.2025	0.0760	0.1896	0.1776	0.0890
	Min.	0.1325	0.1325	0.0000	0.3740	0.3740	0.0000
132 ROR	# Eff. Obs.	20	21	70	132	70	70
Observations	Arith. Mean.	0.8274	0.8202	0.0072	1.0000	0.9905	0.0095
	Stand. Dev.	0.1853	0.1879	0.0170	0.0000	0.0217	0.0217
	Min.	0.2094	0.2094	0.0000	1.0000	0.9036	0.0000

† Li test: exact *p* values are reported in round brackets underneath.

difference between the C-NC and C–C estimates of ITE is $0.7940 - 0.7770 = 0.0170$. A Li test was once again applied to all 192 observations to test the null hypothesis that the two ITE (resp. IMR) distributions are equal. The test statistic obtained is −1.32 (resp. 15.21). The large (resp. small) p value indicates that we do not reject (resp. reject) the null hypothesis that the two ITE (resp. IMR) distributions are equal.

By construction, estimates of ITE obtained using the convexifying DEA estimator (16) can be no higher than the estimates obtained using the asymptotically unbiased estimator (15) (i.e., estimates of ITE obtained using the C–C model can be no higher than those obtained using the C-NC model). Table 2 reveals that, in the case of Chilean hydroelectric power generators, estimates of ITE obtained using (16) are on average 1.70 percentage points lower than those obtained using (15). This represents a $0.017/0.794 = 2.1\%$ decrease in estimated average ITE. For the subset of firms that use the dam (resp. ROR) technology, using the estimator (16) instead of the estimator (15) results in a $0.0385/0.7205 = 5.3\%$ (resp. $0.0072/0.8274 = 0.9\%$) decrease in estimated average ITE. By construction,[17] qualitatively similar conclusions can be drawn regarding the IMRs.

10. Conclusions

In their seminal paper, O'Donnell et al. (2008) consider a metaset that is defined as the union of two or more underlying group-specific sets. They refer to the boundary of the metaset as a metafrontier, and they refer to the boundaries of the group-specific sets as group frontiers. They suggest estimating the metafrontier under the assumption that the metaset is convex. If this assumption is false, then their "convexifying" estimator is biased. Our key result, Proposition 5.5, shows that both C and NC group-specific sets can (and generally do) yield NC metasets. This suggests that the convexification strategy should not be maintained *a priori*. In any case, it should be empirically tested.

We used secondary data on Chilean hydroelectric power plants to explore the consequences of making incorrect assumptions about the convexity or nonconvexity of TPPSs and MTPPS. We focused on the consequences of a convexification strategy for input-oriented estimates of metatechnology ratios and associated measures of technical efficiency. Estimates of input-oriented technical efficiency (ITE), residual input-oriented technical efficiency (RITE) and input-oriented metatechnology ratios (IMRs) were moderately sensitive to these assumptions. While we could have engaged in a data mining exercise to scan for a variety of data sets that would have supported our basic hypotheses more strongly, we have simply taken a data set that is publicly available so that readers can easily duplicate our results. Since one counterexample is sufficient to invalidate a hypothesis, our results can be safely taken to reject the assumption that the convexification strategy suggested by O'Donnell et al. (2008) is empirically innocuous.

We recommend that users of metafrontier methods should form MTPPS as the (possibly non-convex) union of either C or NC TPPSs. The shortcut suggested in the seminal article of O'Donnell et al. (2008) (i.e., convexify the metaset) generally leads to erroneous results. This critical conclusion that convexity need not be an innocuous assumption is in line with some earlier results in the literature. Briec et al. (2004) already illustrated how the measurement of technical and scale efficiencies is affected by the convexity axiom. Kerstens & Managi (2012) show how the Luenberger productivity indicator as well as its decomposition into technical change and efficiency change are affected by convexity. Finally, Briec et al. (2004) establish theoretically, and illustrate empirically, how the convexity axiom not only affects technologies, but also affects the level of the cost function derived from it (see also Balaguer-Coll, Prior & Tortosa-Ausina, 2007 for another empirical illustration).[18] Thus, convexity seems to matter both from a theoretical and empirical perspective.

Furthermore, we can briefly indicate whether and how these results based on general technologies and on a nonparametric approach using data envelopment analysis and free disposal hull estimators can be transposed to alternative frontier methodologies. First, the transposition to alternative nonparametric frontier methods (e.g., conditional C and NC models, C and NC order-m models (see Daraio & Simar, 2007)) is straightforward by definition. Second, adopting this meticulous construction of a MTPPS should be rather straightforward in a deterministic parametric approach: it is just a matter of properly transposing the constructive results from the nonparametric approach. Third, the implications for a proper construction of a MTPPS in the far more popular stochastic frontier model have just recently been explored in Amsler, O'Donnell & Schmidt (2017). Unfortunately, simulation methods must generally be used to construct a stochastic metafrontier that correctly envelops two or more stochastic group frontiers

[17] The RITE estimates obtained using the C–C and C-NC models are identical (both were obtained using the convexifying DEA estimator (14)). If the ITE estimates obtained from the C–C model can be no higher than those obtained from the C-NC model, then the decomposition (9) implies that IMR estimates obtained from the C–C model can be no higher than those obtained from the C-NC model.

[18] In fact, WACM is implicitly based on an FDH technology: see Ray (2004).

(for details, see Amsler et al., 2017). Finally, transposing this construction of a MTPPS in a stochastic nonparametric approach seems rather straightforward (see, e.g., Afsharian, 2017 who focuses on the StoNED method).

Finally, it is clear that further research is needed to assess how the many different metafrontier applications discussed in Section 1 are affected by the possibly incorrect assumption that the MTPPS is convex. We encourage researchers to test this assumption in their empirical work. Some first steps seem to have been taken by Afsharian & Ahn (2015) and Afsharian, Ahn & Harms (2018) when developing some variation on the primal Malmquist productivity index. Furthermore, empirical testing of the impact of assuming an equality in Proposition 5.5 property (c) in contrast to an approach in part (e) remains to be done: this boils down to redoing our empirical analysis using cone technologies. Furthermore, while we have used a rather generally valid test statistic, specific statistical tests of convexity versus non-convexity have been proposed in a theoretical framework developed by Kneip, Simar & Wilson (2016). Implementing such a specific test approach may eventually sharpen our results. In a similar vein, there have been a variety of alternative proposals around to account for heterogeneity in frontier models. Popular methods include the use of latent class models (e.g., Orea & Kumbhakar, 2004 in a stochastic frontier context), or aggregation over groups or industries (e.g., Zelenyuk (2006) or Mayer & Zelenyuk (2014), but see Balk (2016) for some caveats). There are also alternative proposals around to handle heterogeneity: Simar, Vanhems & Van Keilegom (2016) or Tsekouras et al. (2017) to give some examples. In fact, to the best of our knowledge no theoretical or empirical review has ever compared some let alone all of these different methods to account for heterogeneity in a frontier framework. Obviously, there is a lot of scope for future work.

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Appendix A. Proofs of Propositions

Proof of Proposition 5.1.

(a) We first prove that $mon(conv(A)) \subseteq conv(mon(A))$. Let $z \in mon(conv(A))$. Following Definition 5.1, $z = v + (x_+, y_-)$ for some $v \in conv(A)$ and $(x_+, y_-) \in \mathbb{R}^M_+ \times \mathbb{R}^N_-$. Using the notion of C hull, $v = \sum_i \alpha_i u_i$ with $u_i \in A$, $\alpha_i \in \mathbb{R}_+$ and $\sum_i \alpha_i = 1$. Hence, $z = \sum_i \alpha_i u_i + (x_+, y_-) = \sum_i \alpha_i (u_i + (x_+, y_-)) \in conv(mon(A))$. Next, by reversing this reasoning, it can easily be established that $conv(mon(A)) \subseteq mon(conv(A))$, hence yielding the desired result.

(b) We first prove that $cone(conv(A)) \subseteq conv(cone(A))$. Let $z \in cone(conv(A))$. Following Definition 5.1, $z = \lambda v$ for some $v \in conv(A)$ and $\lambda \in \mathbb{R}_{++}$. From the definition of the C hull, it follows that $v = \sum_i \alpha_i u_i$ with $u_i \in A$, $\alpha_i \in \mathbb{R}_+$ and $\sum_i \alpha_i = 1$. Consequently, $z = \lambda \sum_i \alpha_i u_i = \sum_i \alpha_i \lambda u_i \in conv(cone(A))$. Again, this reasoning can be reversed which leads to the desired result.

(c) We first prove that $mon(cone(A)) \subseteq cone(mon(A))$. Let $z \in mon(cone(A))$. Following Definition 5.1, $z = v + (x_+, y_-)$ for some $v \in cone(A)$ and $(x_+, y_-) \in \mathbb{R}^M_+ \times \mathbb{R}^N_-$. Using the notion of conical hull, there exits a $\lambda \in \mathbb{R}_{++}$ and a $u \in A$ such that $v = \lambda u$. Hence, $z = \lambda u + (x_+, y_-) = \lambda\big(u + (\frac{x_+}{\lambda}, \frac{y_-}{\lambda})\big) \in cone(mon(A))$. Next, by reversing this reasoning, it can easily be established that $cone(mon(A)) \subseteq mon(cone(A))$, hence yielding the desired result. □

Proof of Proposition 5.2. The results follow trivially from Definition 5.1. □

Proof of Proposition 5.3.

(a) Since $A \subseteq A \cup B$, preservation of subset relationships results in $mon(A) \subseteq mon(A \cup B)$. By analogy, $mon(B) \subseteq mon(A \cup B)$. Hence, $mon(A) \cup mon(B) \subseteq mon(A \cup B)$. Conversely, if $z \in mon(A \cup B)$, then following Definition 5.1, $z = u + (x_+, y_-)$ for some $u \in A \cup B$ and $(x_+, y_-) \in \mathbb{R}^M_+ \times \mathbb{R}^N_-$. But then, u must be contained in at least one of the sets A or B. Without losing generality, assume that $u \in A$. Then $z \in mon(A) \subseteq mon(A) \cup mon(B)$ which yields the required result.

(b) Since $A \subseteq A \cup B$, preservation of subset relationships results in $cone(A) \subseteq cone(A \cup B)$. By analogy, $cone(B) \subseteq cone(A \cup B)$. Hence, $cone(A) \cup cone(B) \subseteq cone(A \cup B)$. Conversely, if $z \in cone(A \cup B)$, then following Definition 5.1, $z = \lambda u$ for some $u \in A \cup B$ and $\lambda \in \mathbb{R}_{++}$. But then, u must be contained in at least one of the sets A or B. Without losing generality, assume that $u \in A$. Then $z \in cone(A) \subseteq cone(A) \cup cone(B)$ which yields the required result.

(c) Since $A \subseteq A \cup B$, preservation of subset relationships results in $conv(A) \subseteq conv(A \cup B)$. By analogy, $conv(B) \subseteq conv(A \cup B)$. Hence, $conv(A) \cup conv(B) \subseteq conv(A \cup B)$. Clearly, the converse relation no longer holds true. E.g., consider the following counterexample. Consider the subsets $A = \{(1,0,0), (0,1,0)\}$ and $B = \{(1,0,0), (0,0,1)\}$ of $\mathbb{R}^3_+$. Then $conv(A \cup B) = conv(\{(1,0,0), (0,1,0), (0,0,1)\}) = \{(x,y,z) \in \mathbb{R}_+ \mid x + y + z = 1\}$, while $conv(A) = \{(x,y,0) \mid x + y = 1\}$ and $conv(B) = \{(x,0,z) \mid x + z = 1\}$. Clearly, $z = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) \in conv(A \cup B)$, but $z \notin conv(A)$ and $z \notin conv(B)$. □

Proof of Proposition 5.4.

(a) Using Propositions 5.1 and 5.3, $conv(mon(A)) \cup conv(mon(B)) = mon(conv(A)) \cup mon(conv(B)) = mon(conv(A) \cup conv(B))$.
Since the hull operators preserve subset relationships, $conv(A) \cup conv(B) \subseteq conv(A \cup B)$ and $mon(conv(A) \cup conv(B)) \subseteq mon(conv(A \cup B))$. Combination with Proposition 5.1 leads to $conv(mon(A)) \cup conv(mon(B)) \subseteq mon(conv(A \cup B)) = conv(mon(A \cup B))$ which is the required result.

(b) Using Propositions 5.1 and 5.3, $conv(cone(A)) \cup conv(cone(B)) = cone(conv(A)) \cup cone(conv(B)) = cone(conv(A) \cup conv(B))$.
Since the hull operators preserve subset relationships, $conv(A) \cup conv(B) \subseteq conv(A \cup B)$ and $cone(conv(A) \cup conv(B)) \subseteq cone(conv(A \cup B))$. Combination with Proposition 5.1 leads to $conv(cone(A)) \cup conv(cone(B)) \subseteq cone(conv(A \cup B)) = conv(cone(A \cup B))$ which is the required result.

(c) Using Proposition 5.3, $cone(mon(A)) \cup cone(mon(B)) = cone(mon(A) \cup mon(B)) = cone(mon(A \cup B))$ which is the required result. □

Proof of Proposition 5.5. All results follow directly from applying Propositions 5.3 and 5.4. □

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